Mathematical Proofs

Chapter 3 – Sets (Exercise solutions)

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## Section 1: Describing a Set

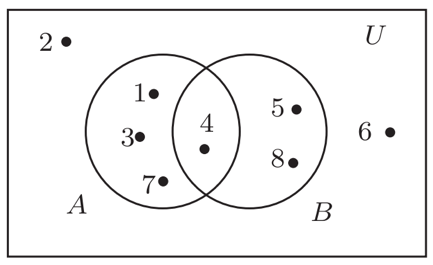
### Exercises

1. Which of the following are sets?
   1. 1, 2, 3 Not a set
   2. {1, 2}, 3 Not a set
   3. {{1}, 2}, 3 Not a set
   4. {1, {2,}, 3} Set
   5. {1, 2, a, b} Set
2. Let . Describe each of the following sets as , where p(x) is some condition on x.
3. Determine the cardinality of each of the following sets:
4. Write each of the following sets by listing its elements within braces.
5. Write each of the following sets in the form , where p(x) is a property concerning x.
6. The set can be described by listing its elements, namely . List the elements of the following sets in a similar manner.
7. The set of even integers can be described by means of a defining condition by . Describe the following sets in a similar manner.
8. Let .
   1. Describe the set A by listing its elements.
   2. Give an example of three elements that belong to B but do not belong to A.
   3. Describe the set C by listing its elements.
   4. Describe the set D in another manner.
   5. Determine the cardinality of the sets A, C and D.
9. For , let and . Determine C.
   1. (because

## Section 2: Subsets

### Exercises

1. Give examples of three sets A, B and C such that
2. Let (a, b) be an open interval of real numbers and let . Describe an open interval I centered at c such that .
   1. Let , then
3. Which of the following sets are equal?
   1. Conclusion: The elements in are equal and C is on its own.
4. For a universal set and two sets and , draw a Venn diagram that represents these sets

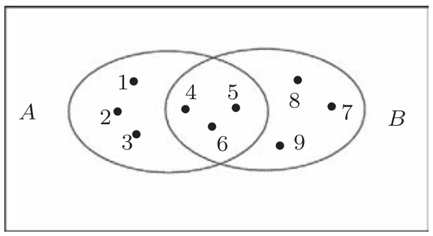


1. Find for
2. Find for .
3. Find and its cardinality.
4. Find and for .
5. For , determine .
6. Give an example of a set S such that
7. Determine whether the following statements are true or false.
   1. If
      1. **False**, e.g.
   2. If A, B and C are sets such that thencan be .
      1. **True**. If , then the cardinality of . Since is a proper subset of C, C must at least have a cardinality of 5.
   3. If a set B has one more element than a set A, then has at least two more elements than.
      1. **False**, if then and (It is true if )
   4. If four sets A, B, C and D are subsets of {1, 2, 3} such that , then at least two of these sets are equal.
      1. **True**. Different combinations of {1, 2, 3} with cardinality 2: . Namely {1, 2}, {1, 3} and {2, 3}.
8. Three subsets A, B and C of have the same cardinality. Furthermore,
   1. 1 belongs to A and B but not to C.
   2. 2 belongs to A and C but not to B.
   3. 3 belongs to A and exactly one of B and C.
   4. 4 belongs to an even number of A, B and C.
   5. 5 belongs to an odd number of A, B and C.
   6. The sums of the elements in two of the sets A, B and C differ by 1.
   7. What is B?

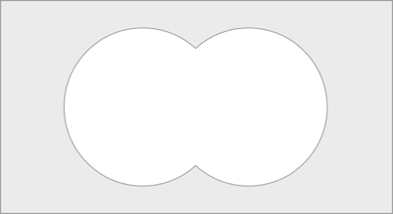
## Section 3: Set Operations

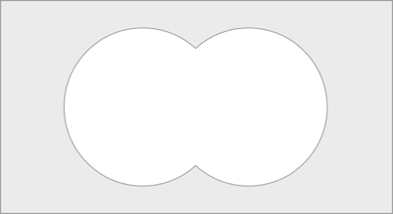
### Exercises

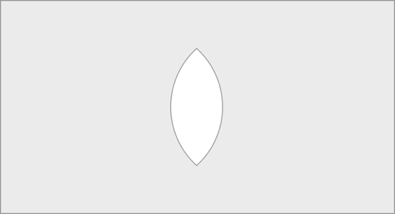
1. Let be the universal set, , and . Determine the following.
2. Give examples of two sets A and B such that . Draw the accompanying Venn diagram.

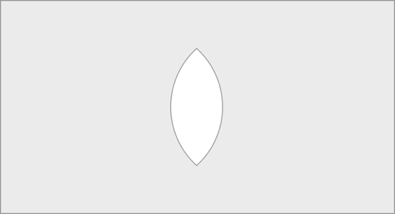


1. Give examples of three sets A, B and C such that but
2. Give examples of three sets A, B and C such that
3. Let U be a universal set and let A and B be two subsets of U. Draw a Venn diagram for each of the following sets.

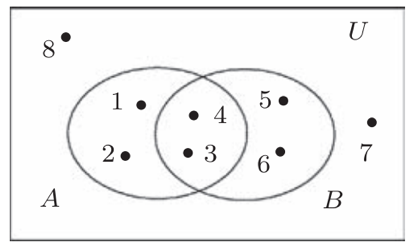




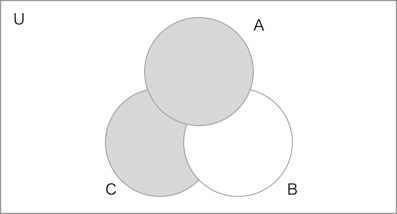


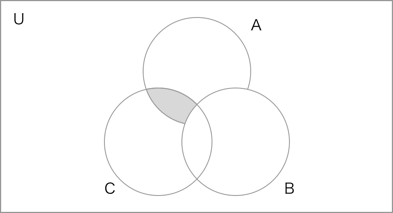


1. Give an example of a universal set U, two sets A and B and accompanying Venn diagram such that



1. Let A, B and C be nonempty subsets of a universal set U. Draw a Venn diagram for each of the following set operations.





1. Let .
   1. Determine which of the following are elements of A:
      1. are elements of A
   2. Determine
   3. Determine which of the following are subsets of A:
      1. are subsets of A

For (d)-(i), determine the indicated sets.

1. Let .
   1. Express A, B and C using interval notation.
2. Give an example of four different sets A, B, C and D such that (1) and and (2) if B and C are interchanged and are interchanged, then we get the same result (.
3. Give an example of four different subsets A, B, C and D of {1, 2, 3, 4} such that all intersections of two subsets are different.
4. Give an example of two nonempty sets A and B such that is the power set of some set.
5. Give examples of two subsets A and B of {1, 2, 3} such that all of the following sets are different: .
   1. Then the different results are:
6. Give examples of a universal set U and sets A, B and C such that each of the following sets contains exactly one element: . Draw the accompanying Venn diagram.
   1. Then the different results are: .

